Fast and Slow, Adiabatic and Geometric Effects in Nonequilibrium Dynamics

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Understanding in the natural sciences often amounts to being able to think together a variety of phenomena in terms of more elementary laws. At the same time, phenomena appear on certain scales of description, and even though certain sets of phenomena and descriptions appear autonomous we hope to be able to move between different scales and to derive one level of description from another one. Statistical mechanics is a standard example of such a transfer science; thermodynamic and hydrodynamic behavior is derived and corrected by including fluctuations on the level of the more microscopic dynamics. That is at least working well for equilibrium statistical mechanics or when local equilibrium is established. Big successes include then the understanding of phase transitions and critical phenomena within a common framework of Gibbs ensembles. The renormalization group idea makes it also clear that many details of the microscopic model do not matter so much.

The construction of nonequilibrium statistical mechanics requires important dynamical considerations and purely thermodynamic factors as received in terms of entropy and free energy functionals probably do not suffice. Within these dynamical features, various scales of description again emerge, involving static or quasi-static to very fast degrees of freedom. A great deal of approximation techniques are in fact based on separation of time scales, such as in adiabatic theorems, Born–Oppenheimer approximation etc. The workshop revisited various aspects of these ideas, and in particular within the context of nonequilibrium statistical mechanics.

Karel Netočný (Praga) gave a talk on quasi-static heat processes in small non-equilibrium systems. Quasi-static evolutions give rise to geometric representations, e.g. for nonequilibrium heat capacities. The same topic can also be discussed in a quantum context. Joseph Avron (Haifa) discussed the geometry of quantum transport of open quantum systems. These open quantum sys-
tems are described by Lindblad evolutions and a precise notion of quasi-static and adiabatic evolution is possible. That was the subject of Gian-Michele Graf (Zürich) talking about adiabatic evolution and dephasing.

A major theme of nonequilibrium statistical mechanics is of course the derivation of effective dynamics on a more coarse grained level of description from microscopic dynamics. A major goal is the derivation of irreversible differential equations, such as Fourier’s law, from the laws of classical or quantum mechanics. That was the theme in the talk of Wocjiech De Roeck (Heidelberg), showing a derivation of diffusion in Hamiltonian quantum systems. Another change of level of description occurs in the derivation of reaction rate equations from diffusion equations. That goes back to the work of Kramers, for which Nils Berglund (Orléans) gave a review of the Kramers law concerning validity, derivations and generalizations. The derivation itself of diffusion processes from microscopic dynamics was the subject of the talk of Robert MacKay (Warwick), in a derivation of the Langevin equation for slow degrees of freedom of Hamiltonian systems. Finally, Senya Shlosman (Marseille) considered a mean field approach to global dynamics with the explanation of spontaneous resonances and multiple steady states in queuing networks.
Abstracts of the talks on “Fast and slow, adiabatic and geometric effects in nonequilibrium dynamics”, January 26, 2011

Yosi Avron (Haifa): Geometry of quantum transport of open quantum systems

I shall describe a geometric theory of adiabatic transport in driven open systems governed by dephasing Lindbladians. The coefficients of dissipative transport are determined by the Fubini–Study metric and the coefficients of non-dissipative transport by the adiabatic curvature. This gives a possible mechanism for residual resistance for gapped systems at low temperatures. When the metric and symplectic form are compatible non-dissipative terms in the inverse matrix of transport coefficients are immune to dephasing. Three examples of compatible systems are: The qubit, coherent states for the Harmonic oscillator and the lowest Landau level on a torus. Based on joint work with M. Fraas, G.M. Graf, and O. Kenneth.

Nils Berglund (Orleans): The Kramers law – validity, derivations and generalizations

The Kramers law describes the mean transition time of an overdamped Brownian particle between local minima of a multiwell potential landscape. Though the law has been conjectured more than 70 years ago, the first full proof of its validity, up to multiplicative errors going to one in the zero-noise limit, appeared only in 2004. We plan to outline the main ideas of this proof and other approaches, and provide generalizations to potentials with nonquadratic saddles and stochastic partial differential equations. Partly based on joint work with Florent Barret, Bastien Fernandez and Barbara Gentz.

Gian Michele Graf (Zurich): Adiabatic evolution and dephasing

Lindbladians are generators of the effective dynamics of open quantum systems. We focus on dephasing Lindbladians. Like Hamiltonians of isolated quantum mechanical system, but in contrast to generic Lindbladians, they exhibit several stationary states.

The adiabatic evolution of an isolated quantum mechanical system exhibits no irreversible transitions if its Hamiltonian undergoes a slow, transient time-dependence. By contrast, if a dephasing Lindbladian undergoes such a change, transitions are typically irreversible. I’ll present a formulation of the adiabatic theorem which accounts for the different kinds of transition, though treating Hamiltonian and Lindbladian dynamics on equal footing. We’ll then present two applications.

The first one is the solution of an optimization problem. Given are a path of dephasing Lindbladians, two states (“base” and “target”) which are stationary w.r.t. the corresponding endpoint, and an amount of time to be spent. Sought is time schedule to be used on the path in order to bring the evolved “base” as close as possible to the “target”.
In a second application we apply the result to transport and linear response theory. In the context of dephasing Lindbladians, the coefficients of dissipative conductance are determined by the Fubini–Study metric, while their non-dissipative counterparts are determined by the adiabatic curvature. If the metric and the (symplectic) curvature form are compatible, in the sense of defining a Kähler structure, then the non-dissipative resistance coefficients are immune to dephasing. We give some examples of compatible systems. (Joint work with Y. Avron, M. Fraas, P. Grech, and O. Kenneth).

Robert MacKay (Warwick): Langevin equation for slow degrees of freedom of Hamiltonian systems

A way is sketched to derive a Langevin equation for the slow degrees of freedom of a Hamiltonian system whose fast ones are mixing Anosov. It uses the Anosov–Kasuga adiabatic invariant, martingale theory, Ruelle’s formula for weakly non-autonomous SRB measures, and large deviation theory.

Christian Maes (Leuven): Corrections to Archimedes’ law in granular media and other statistical forces

Statistical forces arise as the influence of internal or residual degrees of freedom after some scale separation. These forces are mostly known and studied under equilibrium conditions. We see what changes must be taken into account when dealing with nonequilibrium systems.

Karel Netočný (Praha): Quasistatic heat processes in small non-equilibrium systems

We give a survey of some new approaches towards better understanding the energy exchange between a slowly time-dependent system and its surroundings. It will be explained via examples of stochastic systems driven by non-gradient force, how the leading non-divergent part in the adiabatic expansion for heat becomes ‘geometrical’, and what are the main differences from the familiar framework of equilibrium thermodynamics. We will discuss some difficulties in constructing meaningful non-equilibrium generalizations of heat capacity and thermodynamic potentials. (Collaborators: E. Boksenbojm, C. Maes, J. Pesek, and B. Wynants).

Wojciech De Roeck (Heidelberg): Diffusion in Hamiltonian quantum systems

We discuss recent work concerning the long-time behaviour of a quantum particle coupled to phonons at positive temperature. Under some conditions, we prove diffusion for such a system. The main assumption is that the particle has an internal degree of freedom and a huge mass. The analysis proceeds via distinguishing a fast and slow timescale. On the fast timescale, the internal degree of freedom thermalizes under influence of the photons. This makes the system ‘effectively stochastic’ for longer times. Then, on the slow timescale, we
observe diffusion, the study of which is greatly simplified by the fact that the system is effectively stochastic.

**Senya Shlosman (Marseille): Spontaneous resonances and multiple steady states in the queuing networks**

We study particle systems corresponding to highly connected queuing networks. We examine the validity of the so-called Poisson Hypothesis (PH), which predicts that the Markov process, describing the evolution of such particle system, started from a reasonable initial state, approaches the equilibrium in time independent of the size of the network.

This is indeed the case in many situations. However, there are networks for which the relaxation process slows down. This behavior reflects the fact that the corresponding infinite system undergoes a phase transition. It is characterized by the property that different nodes of the network start to evolve in a synchronous way.

Such transition can happen only when the load per node exceeds some critical value, while in the low load situation the PH behavior holds. The load thus plays the same role as the inverse temperature in statistical mechanics.

We will discuss a related open problem of ergodicity of interacting particle systems with unique stationary state.